JOURNAL OF BIOMECHANICS

Journal of Biomechanics 41 (2008) 488–489

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Erratum

Erratum "A new non-orthogonal decomposition method to determine effective torques for three-dimensional joint rotation" [J. Biomech. 40 (2007) 871–882]

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We regret to inform that the non-orthogonal decomposition method proposed in Hirashima et al. (2007) is insufficient for the purpose of understanding an effect of the torques on the joint angular accelerations in 3D multi-joint movements. The inappropriate parts are 2.4.1, 2.4.2, and 2.4.3. We replace these parts by the following.

Appropriate method

We can obtain the three equations of motion[1] (Eqs. 15, 19, 20 in Hirashima et al. (2007)) that are expressed by the joint angular accelerations of the shoulder ($\ddot{\boldsymbol{\theta}}_1$), elbow ($\ddot{\boldsymbol{\theta}}_2$), and wrist ($\ddot{\boldsymbol{\theta}}_3$). They can be simply written as follows:

$$\begin{cases} \boldsymbol{I}_{11}\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{12}\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{13}\ddot{\boldsymbol{\theta}}_3 = \boldsymbol{\tau}_1 + \boldsymbol{V}_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}_1(\boldsymbol{\theta}), \\ \boldsymbol{I}_{21}\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{22}\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{23}\ddot{\boldsymbol{\theta}}_3 = \boldsymbol{\tau}_2 + \boldsymbol{V}_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}_2(\boldsymbol{\theta}), \\ \boldsymbol{I}_{31}\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{32}\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{33}\ddot{\boldsymbol{\theta}}_3 = \boldsymbol{\tau}_3 + \boldsymbol{V}_3(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}_3(\boldsymbol{\theta}), \end{cases} \tag{1}$$

where $\boldsymbol{\tau}_i$ is the resultant joint torque (RJT) at the joint i. The RJT includes not only the active muscle torque but also the passive torque from viscoelastic elements such as ligaments, bones, and other connective tissues. $\boldsymbol{V}_i(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ is the velocity-dependent torque (VDT) at the joint i. $\boldsymbol{g}_i(\boldsymbol{\theta})$ is the gravity torque (GRA) at the joint i. Note that these vectors are expressed in terms of global coordinate system. To express them in terms of each joint coordinate system, we multiplied the coordinate transformation matrix[2] (${}^1\boldsymbol{R}$,${}^2\boldsymbol{R}$, and ${}^3\boldsymbol{R}$) for each equation as follows:

$$\begin{cases} {}^1\boldsymbol{R}(\boldsymbol{I}_{11}\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{12}\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{13}\ddot{\boldsymbol{\theta}}_3) = {}^1\boldsymbol{R}(\boldsymbol{\tau}_1 + \boldsymbol{V}_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}_1(\boldsymbol{\theta})), \\ {}^2\boldsymbol{R}(\boldsymbol{I}_{21}\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{22}\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{23}\ddot{\boldsymbol{\theta}}_3) = {}^2\boldsymbol{R}(\boldsymbol{\tau}_2 + \boldsymbol{V}_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}_2(\boldsymbol{\theta})), \\ {}^3\boldsymbol{R}(\boldsymbol{I}_{31}\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{32}\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{33}\ddot{\boldsymbol{\theta}}_3) = {}^3\boldsymbol{R}(\boldsymbol{\tau}_3 + \boldsymbol{V}_3(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}_3(\boldsymbol{\theta})). \end{cases} \tag{2}$$

We expressed the angular acceleration vectors and torque vectors in terms of each joint coordinate system by using the upper-left subscript[3]:

$$\begin{cases} {}^1\boldsymbol{R}(\boldsymbol{I}_{11}\,{}_1\boldsymbol{R}\,{}^1\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{12}\,{}_2\boldsymbol{R}\,{}^2\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{13}\,{}_3\boldsymbol{R}\,{}^3\ddot{\boldsymbol{\theta}}_3) = {}^1\boldsymbol{\tau}_1 + {}^1\boldsymbol{V}_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + {}^1\boldsymbol{g}_1(\boldsymbol{\theta}), \\ {}^2\boldsymbol{R}(\boldsymbol{I}_{21}\,{}_1\boldsymbol{R}\,{}^1\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{22}\,{}_2\boldsymbol{R}\,{}^2\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{23}\,{}_3\boldsymbol{R}\,{}^3\ddot{\boldsymbol{\theta}}_3) = {}^2\boldsymbol{\tau}_2 + {}^2\boldsymbol{V}_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + {}^2\boldsymbol{g}_2(\boldsymbol{\theta}), \\ {}^3\boldsymbol{R}(\boldsymbol{I}_{31}\,{}_1\boldsymbol{R}\,{}^1\ddot{\boldsymbol{\theta}}_1 + \boldsymbol{I}_{32}\,{}_2\boldsymbol{R}\,{}^2\ddot{\boldsymbol{\theta}}_2 + \boldsymbol{I}_{33}\,{}_3\boldsymbol{R}\,{}^3\ddot{\boldsymbol{\theta}}_3) = {}^3\boldsymbol{\tau}_3 + {}^3\boldsymbol{V}_3(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + {}^3\boldsymbol{g}_3(\boldsymbol{\theta}), \end{cases} \tag{3}$$

where ${}^i\boldsymbol{\tau}_i$ is the RJT at the ith joint that is expressed in the ith joint coordinate system. For example, ${}^1\boldsymbol{\tau}_1 = (\tau_{1-\mathrm{IE}} \quad \tau_{1-\mathrm{ED}} \quad \tau_{1-k})^{\mathrm{T}}$consists of three RJTs about the internal/external axis, elevation/depression axis, and third axis at the shoulder. Similarly ${}^i\ddot{\boldsymbol{\theta}}_i$ is the ith joint angular acceleration vector expressed in the ith joint coordinate system. For example, ${}^1\ddot{\boldsymbol{\theta}}_1 = (\ddot{\theta}_{1-\mathrm{IE}} \quad \ddot{\theta}_{1-\mathrm{ED}} \quad \ddot{\theta}_{1-k})^{\mathrm{T}}$. Eq. (3)

DOI of original article: 10.1016/j.jbiomech.2006.03.012

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[1]Please note that the equations of motion (Eqs. 15, 19, 20 in Hirashima et al. (2007)) themselves are valid.

[2]The matrix ${}^i\boldsymbol{R}$ indicates a coordinate transformation matrix from the global coordinate system to the joint coordinate system i. ${}_i\boldsymbol{R}$ indicates a coordinate transformation matrix from the joint coordinate system i to the global coordinate system.

[3]Upper-left subscript indicates the coordinate system in terms of which the vector is expressed. When a vector is expressed in terms of the global coordinate system, upper-left subscript is omitted.

0021-9290/$ - see front matter
doi:10.1016/j.jbiomech.2007.11.013

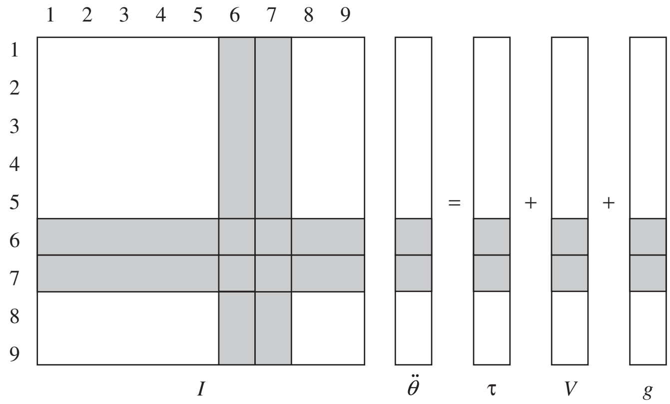

Fig. 1. Gray areas are eliminated to obtain the 7-DOF equation of motion.

can be expressed by a matrix form as follows:

$$\boldsymbol{I}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \boldsymbol{\tau} + \boldsymbol{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta}), \tag{4}$$

where

$$\boldsymbol{I}(\boldsymbol{\theta}) = \begin{pmatrix} {}^{1}\boldsymbol{R}\boldsymbol{I}_{11\,1}\boldsymbol{R} & {}^{1}\boldsymbol{R}\boldsymbol{I}_{12\,2}\boldsymbol{R} & {}^{1}\boldsymbol{R}\boldsymbol{I}_{13\,3}\boldsymbol{R} \\ {}^{2}\boldsymbol{R}\boldsymbol{I}_{21\,1}\boldsymbol{R} & {}^{2}\boldsymbol{R}\boldsymbol{I}_{22\,2}\boldsymbol{R} & {}^{2}\boldsymbol{R}\boldsymbol{I}_{23\,3}\boldsymbol{R} \\ {}^{3}\boldsymbol{R}\boldsymbol{I}_{31\,1}\boldsymbol{R} & {}^{3}\boldsymbol{R}\boldsymbol{I}_{32\,2}\boldsymbol{R} & {}^{3}\boldsymbol{R}\boldsymbol{I}_{33\,3}\boldsymbol{R} \end{pmatrix},$$

$$\ddot{\boldsymbol{\theta}} = \begin{pmatrix} {}^{1}\ddot{\boldsymbol{\theta}}_1 \\ {}^{2}\ddot{\boldsymbol{\theta}}_2 \\ {}^{3}\ddot{\boldsymbol{\theta}}_3 \end{pmatrix}, \quad \boldsymbol{\tau} = \begin{pmatrix} {}^{1}\boldsymbol{\tau}_1 \\ {}^{2}\boldsymbol{\tau}_2 \\ {}^{3}\boldsymbol{\tau}_3 \end{pmatrix},$$

$$\boldsymbol{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{pmatrix} {}^{1}\boldsymbol{V}_1(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ {}^{2}\boldsymbol{V}_2(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \\ {}^{3}\boldsymbol{V}_3(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \end{pmatrix}, \quad \boldsymbol{g}(\boldsymbol{\theta}) = \begin{pmatrix} {}^{1}\boldsymbol{g}_1(\boldsymbol{\theta}) \\ {}^{2}\boldsymbol{g}_2(\boldsymbol{\theta}) \\ {}^{3}\boldsymbol{g}_3(\boldsymbol{\theta}) \end{pmatrix}.$$

Note that the system mass matrix ($\boldsymbol{I}(\boldsymbol{\theta})$) is now a 9×9 matrix. Actually, however, the human upper extremity has only 7 DOFs. Therefore, we eliminated the 2 DOFs (i.e., varus/valgus at the elbow and the longitudinal rotation at the wrist) as shown in Fig. 1 and obtained the 7-DOF equation of motion that can be expressed in the same form as Eq. (4).

To understand the effect of the torques on the joint angular accelerations, inertial property of the kinematic chain must be taken into account. Therefore we calculated the angular acceleration caused by the torques by multiplying the inverse of the $\boldsymbol{I}(\boldsymbol{\theta}) \in R^{7\times 7}$ as follows:

$$\ddot{\boldsymbol{\theta}} = \boldsymbol{I}(\boldsymbol{\theta})^{-1}(\boldsymbol{\tau} + \boldsymbol{V}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta})). \tag{5}$$

This equation, for example, tells us that the angular acceleration of shoulder internal rotation ($\ddot{\theta}_1$) is produced by (1) all the 7 RJTs, (2) VDTs, and (3) gravity torques.

$$\begin{aligned} \ddot{\theta}_{1-\mathrm{IE}} &= \sum_{i=1}^{7} A_{1i}\tau_i + \sum_{i=1}^{7} A_{1i}V_i + \sum_{i=1}^{7} A_{1i}g_i \\ &= (\ddot{\theta}^{\tau1}_{1-\mathrm{IE}} + \ddot{\theta}^{\tau2}_{1-\mathrm{IE}} + \cdots + \ddot{\theta}^{\tau7}_{1-\mathrm{IE}}) + \ddot{\theta}^{V}_{1-\mathrm{IE}} + \ddot{\theta}^{g}_{1-\mathrm{IE}}, \end{aligned} \tag{6}$$

where A_{1i} is the (1, i) component of the matrix $\boldsymbol{I}(\boldsymbol{\theta})^{-1}$.

In the supplementary material, we show how an angular acceleration of each DOF is produced by these torque components during a pure shoulder internal–external rotation movement.

Lastly we pointed out the simple typo in Eq. (16) in Hirashima et al. (2007). $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, and $\boldsymbol{C}_3$ must be as follows:

$$\boldsymbol{C}_1 = \boldsymbol{\omega}_1 \times (\boldsymbol{I}_1\boldsymbol{\omega}_1),$$
$$\boldsymbol{C}_2 = \boldsymbol{\omega}_2 \times (\boldsymbol{I}_2\boldsymbol{\omega}_2),$$
$$\boldsymbol{C}_3 = \boldsymbol{\omega}_3 \times (\boldsymbol{I}_3\boldsymbol{\omega}_3).$$

Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.jbiomech.2007.11.013.

Reference

Hirashima, M., Kudo, K., Ohtsuki, T., 2007. A new non-orthogonal decomposition method to determine effective torques for three-dimensional joint rotation. Journal of Biomechanics 40, 871–882.